Муниципальное казенное учреждение
«Центр поддержки образования»
муниципального образования Динской район

Сборник №5
по подготовке к ГИА по МАТЕМАТИКЕ по теме «Элементы теории вероятностей»

K h k l Z \ b v l _ m e q b l _ e v f Z l _ f Z l b d b ; H M K H R «
l m l b e b @ Z Z g g _ l l Z G b d h e Z _ \ g Z

k l > b g k d Z y

Эксперимент beb hiu_ex^Z_y \ gZ[ex^_gbbl_afZ_o h[e_dy\e_gbyf kljh]h hij_^_e_gguo mkeh\byo b baf_j_gbb agZq_gbc aZjZ wlbo h[t_dlh\ y\e_gbc Wdki_jbf_gl gZfau_Zxeb khZlbfkl\bq_k ih\lhj_gijZdlbq_kdb g_baf_gguo _hZjeh\byogh_ qbkeh jZa ih^[fhg_lu

Исходом эксперимента gZau\lZx agZq_gb_ gZ[ex^Z_afhZdZ g_ihkj_^kl_ggh ihemq_ggh_ ih hdhgqZgbb wdki_jbf_glZ aZdZgqb\Z_lky h^gbf b lhevdh h^gbf bkoh^hf

Событием gZ[ex^Z_fuf \ wdki_jbfZgu\Zxl ihy\e_gb_ h[eZ^Zxs_]h aZjZg__ mdZaZgguf k\hckl\hf

;jhkZ_f r_klb]jZgguc b]jZevguc dm[bd Mkeh\by ih_joghkhvkmlkl\b_ kbevguo \g_cgkb\bc\hadjhf_ ijbly`\_gby A ^\b`_gby jmdb [jhkZxs_Bkodm^ubdwdki_jbfgbfZj]jZgb dm[b hdZaZ\r_cky k_jom ihke_ _]h hklZgh\db \hafh`gh jZaguo

Ke_^m_l ohjhrh hlebqZlv kh[ulbyKhel_bkohh^hZ\d`\_ bf\_lv \ \ dhgdj\_lghf wdki\_jbf\_gl\_ fh]ml ihy\eylvky g\_ eZxdb\_ kdhf[uhbu fh]ml [ulv hij\_^\_e\_gu q\_j\_a k\hckl\Z bkoh^h\ wlh]h wdki\_jbf \hafh`guo \ ^Zgghf wdki_jbf_ijhlba\hevgh Zkyhij_keyf_bkoh wdki_jbf_glZ

KemqZcguf hkbl_evgh dhfie_dkZ mkeh\bc kh[ulb_ dhlh hkms_kl\e_gbb mdZaZggh]h dhfie_dkZ mkeh\bc fh`_l eb[h i L_hjby _jhylghkl_c bf__l ^_eh kh kemqZcguf hbZkhf[ulbfyf_b ij_^kdZaZlvbahij\ eb _^bgbqgh_ kh[ulb_ beb g_l L_hjby _

1. Основные понятия теории вероятностей

Некоторые формулы комбинаторики

Dhf[bgZpbb khklhysb_ ba h^ghc bijZa_gbukhwed_nighhk\ jZaebqZxsb_ky lhevdh ihjy^dhf bo jZkihe_hj_kdZyh\gdZfbkZlk \k_o \hafh`guo i_j_klZgh\hd hij_^_ey_lky ijhba_^i_gb_f qbk

$J_Q = 1 \cdot 2 \cdot 3 \cdots n = n!$

Dhf[bgZpbbweh_f_glh\ khklZ\e_jZaebqbguo wh_f≤g hlebqZxsb_ky ^jm] hl ^jm]Z eb[h we_f_glZfb eb[h bo jZaf_s_gbyQfbkeh \k_\hafh`gsu_ogbZaf_

$A_n^m = n(n-1)(n-2)\cdots(n-m+1)$

Dhf[bgZpbb kh^j`Zxsb\_ghh\ dZ`^Zy khklZjZea_egbguguo we_f_glh\

≤ b jZaebqZxsb_ky ohly [fu_ghhfgbfgZvæu\Zxlky Qbkeh khq_lZgbc hij_^_ey_lky nhjfmehc

$C_n^m = \frac{n!}{m!(n-m)!}$, $C_n^m = C_n^{n-m}$, $A_n^m = P_m \cdot C_n^m$

< qZklghklb \lhjmx ba nhjfme m^h[gh bkihevaQ\Zlv \ jZl
ljbf_j Kdhevdbfb kihkh[Zfb fh`gh \u[jZlv Z ih rZjZ
dhlhjhc gZoh^ylky rZjh\ [ih rZjZ ba dhZaho^ylky dh
rZjh\"

Решение. Bkdhfh_ qbkeh kihkh[h\
Z $C_{36}^2 = \frac{36!}{2!34!} = \frac{35 \cdot 36}{2} = 630$, б) $C_{36}^{32} = C_{36}^4 = \frac{36!}{4!32!}$

2. Виды случайных событий

Определение 1 Kh[ulby gZag_Zkl\f_klguflkeb \ h^ghf b lhf
bkiulZgbb ihyhe_gbh ba gbo bkdexqZ_l ihy\e_gb_ ^jm]bo
©hjeZª ijb ih^[jZku\Zgbb dfxhqZ_l ihy\e_gb_ \ wlhf `_ bkiulZgl
gZh[hjhl

Определение 2. G_kdhevdh kh[ulbc ihejZax\jm_ikeb \ j_amevl
bkiulZgby ihy\e_gb_ h^ghyh ba gbo yley_jkuf ^khk[ulb_f GZijb
ijb ijhba_^_gbb \uklj_egZ ihbkfhurlZgb_ h[yaZl_evgh [m^_l e
eb[h ijhfZo wlb ^\Z kh[ulby h[jZamxl ihegmx]jmiim
lhgylb_ _jhylghklb

GZah_f dZ`^uc ba \hafh`guo j_amevelZfl_hgZ[ujkhZgby_f
beb L_ we_f_glZjgu_ bkoh^u jhkhmxu_ gZbkgl_gZau\Z
[eZ]hijbylgukfh[ulbyfb

Определение 3. Hlghr_gb_ qbkeZ [eZ]hijbylkl\mxwe_ofkl[Zjbxc
bkoh^m\d h[s_fm qbkem jZ\gh\hafh`guo ye_kf\fl_lZljguo nbkoh
h[jZamxsbo ihegmx]jmiim_jhylghklb :

J : PQ

<_jhylghklv kh[ulhby hagZqZJ_lkylhgylb_ _jhylgeykl_bkylh^gb
ba hkgh\guo \ l_hjbb \>jZhoydbhkjZg__ _]h hij_^_e_gb_ y\ey_l
Ba g_]h \ul_dZxl g_dhlhju_ k\hckl\Z

Свойство 1. <_jhylghklv ^hklh_jgh]h kh[ulby jZ\gZ _^bgbp_

Свойство 2. <_jhylghklv g_\hafh`gh]h kh[ulby jZ\gZ gmex

Свойство 3. <_jhylghklv kemqZcgh]h hkeh[ublbyevkghv_iqbkeh J
Ke_^h\Zl_evgh _jhylghklv ex[h]h kh[ulbykhmm^h\e_l\hjy_
≤ J :≤
ljbf_j < dhjh[d_ e_`Zl rZjh\ [_euo b q_yggukhkl\GZc
lh]h qlh ba iylb \ayluo gZm]Z^ rZjh\ [m^_l [_euo

Решение. GZc^_f qbkeh [eZ]hijbylguo bkoh^h\lhgjbkeh fhk ghk
\aylv [_euo rZjZ ba bf_xsboky jZ\gh

$$C_6^4 = \frac{6!}{2!\,4!} = 15$$

H[s__ qbkeh bkoh^h\ hij_^_ey_lky qbkehf kC$_{16}^5$ = 252 gbc
Kh]eZkgh hij_^_e_gbx bkdhfZy _jhylgh8lP Q

3. Умножение вероятностей

3.1. Произведение событий и условная вероятность.

Определение 4. ljhba_^_gb_fo kh[ulbbc gZau\Z_lkylbk_h[u
hagZqZxksh\f_klgh_ ihy\e_gb_ wlbo kh[ulbc

?keb ijb \uqbke_gbb _jhylghklb kh[ulby gdp_dgZbdbo d^jhf
g_h[oh^bfh]h dhfie_dkZ gmkeghZ\bZ]Z_lky lh lZdZy _jhylgh
[_amkeh\ghc

?keb `_ gZeZ]Zxlky ^jm]b_ ^hihegbh^e\gZus_b_mkkeh\ghc
kh[ulby lh \ghjkhlwlZdh]h kh[ulbymkZeahu\gZh_dky

Определение <_jhylghklv kh[ulby^iheh`_gbb h gZebqbb k
gZau\Zmxkeh\ghc _jhylghklvx

Ijbf_j < ysbd_ e_`Zl ^_lZe_c ba gbo g_kslZdZZjl
^\Z`^u [_jml ih h^ghc ^_lZeb g_ \ha\jZsZy \b_jhy[jgZhkhv lG[Zhc
qlh \h \lhjhc jZa ba ysbdZ [m^_l ba\e_q_gZ² kklZ[u\Z[lgZkeb^\lZ
i_j\uc jZa \ayeb^Zjlkhlmgx

Решение. Ihke_ i_j\h]h ba\e_q_gby \ ysbdf__blky ^klZZge^_cjlg
b ke_^h\Zl_evgh bkdhfZy _jhylghklv

J: <

Imklv l_i_jv ba_klgu _jhykhg[ukbyb mkeh\gZyylg\hkjhv
kh[ulbyLh]^Z kijZ_^eb\Z ke_^mxsZy l_hj_fZ

Теорема 1. <_jhylghklv ijhba_^_gby ^jm^o_ky[ukbycnhijfmehc

J :< J :<

< l_hjbb ^hdZau\Z_lky qlh kijZ_^eb\h jZ_gkl\h

AZf_qZgb_

Ijbf_j < mkeh\byo ijbfg_ZZlb _jhylghklv lh]h \ulh j\Za_j
ba\e_q_gZ g_klZg^ZjlgZy ^_lZev² klZhg\Zhgjhc jZa

Решение. BlZd kh[ulb_wlh ba\e_q_gb_ ba ysbdZ g_klZg^Z
kh[ulb_klZg^Zjlghc Lh]^Z _jhylghklvZ mkeh\gZy_jhylghklv

Bkdhfzy _jhylghklv ijhba_^_gby wlbo kh[ulbc bo k
mdZaZgghf ihjy^d_ **теоремы 1**

. ≈

Ijbf_j < mjg_ gZoh^ylky [_euo rZjZ djZkguc^oZqbm
ba\edZxl ih h^ghfm rZjm g_ \ha\jZsZy _]h h[jZlgh GZclb _
i_j\uc jZa ihy\blky [_euc rZj: kh[ulhjbc ?ZdjZkguc kh[ulb_
lj_lbc² kbgbc kh[klb_

Решение. <_jhylghklv ihy\e_gby [_eh]h rZjZ \ i_j\hf ba\e_
mkeh\gZy _jhylghklv ihy\e_gby djZkgh]h rZjZ \h \lhjhf
ihy\e_gby \ i_j\uc jZa [_eh]h rZjZ

mkeh\gZy _jhylghklv ihy\e_gby kbg_]h brZjZ
ijb mkeh\byo ihy\e_gby \ ij_^u^msb]hbbad\eZkqg_hglbyroZj[h\eh
Bkdhfzy _jhylghklv hij_^_ey_lhkyqi_hlmfhjqmhe_kh[ulbc [hevQ_ ^\

K ≈

G_aZ\bkbfu_ kh[ulby

Определение 6. Kh[ulb_gZau\Z_glkayZ\bkbfufkh[ulby_keb mkeh\g
_jhylghklv kh[uZlbgyZ _]h [_amkehlgghkb_ijhy\e_gb_ kgh[ulbby_l
gZ _jhylghklv

kh[ulby

>eyg_aZ\bkbfuo kh[ulbc l_hj_fZ mfgh`_gby _jhylghkl_c
bf__l \b^

J : : :⋯: Q J :J : J :⋯J :Q i!

JZ_gkl\h ijbgbfZ_lky aZ hij_^_e_gb_ g_aZ\bkbfuo kh

Ijbf_j GZclb _jhylghklv ihjZ`_gbyb pkheb_klghc k[lj_ev
lj_fy hjm^byfb _keb _jhylghklb ihjZ`_gby pu_eb hjm^bybfb
khhl_lkl_ggh kh:[ukbyb K

Решение. lhkdhevdm kh[ulbyb y\eyxlky g_aZ\bkbfudlhfZyh _jhylghklv \uqbkey_lky kh]eZkjgQh nhjfme_

. .

Dh]^Z \ j_amevlZl_ bkiulZgby fh]mgl_aZ\bkbfudlhfZyh k ba_klgufb _jhylghklyfb bo ihy\e_gby hkh[uc bgl_j_k ij_^kl _jhylghklb gZkImie_gby ohly [u h^gh]h ba gbo gZ?jbgfZ_glb\ k _jhylghklv gZkImie_gby eb[h h^gh]h eb[h ^\mo efb[vlhlj_ kh[ulby q_jK_iaZ_^eb\Z ke_^mxsZy l_hj_fZ

Теорема 2. <_jhylghklv ihy\e_gby ohly [u hf^ugh]hhbualbgc_aZ\bk hij_^_ey_lky nhjfmehc

$$P(A) = 1 - g_1 g_2 g_3 \dots g_n$$

]^_ ² _jhylghklb kkhlhm\x_sbo ijhlb\hiheh[ùghuc$hlh Q

< qZkIghf kemqZ_ dh]^Z bkf_xkhh[ubgyZdh\mxgh\kjvhyla nhjfmeu ke_^m_l 3q$h T^Q T S **(7)**

Ijbf_j GZ i_j_\hadm gZjimZ\Ze_gu Z\lhfh[bey <_jhy gZoh`^_gby dZ`^hc ba fZrbg \ bkijZ\ghf khkIhlyg\b_jhjyZl\gghZkIv lh]h qlh \ jZ[hl_ mqZkI\m_l ohly [ughu^ogbyawlu^]heZylhfh[be_

Решение. <_jhylghklv ijhlb\hiheh`gh]h kh[ulbyZ\gZZrjbZg\ZZg_bk lh nhjfme_ gZoh^byflgbkkdlhvfmjko _jh

$ ±

О классическом, статистическом и геометрическом определениях вероятности

LjZ^bpbhggh_ baeh`_gh **теории вероятностей включает в себя три «определения» вероятности:** deZkqk_bkdh_hlghr_gb_ qbkeZ [eZ]hijbylk bkoh^h\ d h[s_fm qbkemkIZlabfkl`gqu_okdihj_^_e hlghkbl_evghc c ijb g_h]jZgbq_gghf m_ebq_gbb q[_keZ_lbkqu_kZdhlgchr_bgb_ ^eb iehsZ^_c beb h[t_fh\ ^\mos_hqeZkd_hchjhc e_`Zl \k_ \hafh`gu_ [eZ]hijbylks_mcx \ dhlhjhc e_`Zl bgl_j_kmxsb_ gZk bkoh^u

<_jhylghklwlh f_jZ \hafh`ghklb ihy\e_gby kemqZhggZh]h^kZ b dZ`^h_ kemqZcgh_ kkhlhu_ba_gZbq_glb_ _ebqbgm wlhc _jhylg fh`gh kjZ\gblv k ^ebghc iehsZ^vx beb h[t_fhf GZijbf_j bf__l hij_^_e_ggmx ijhkljZgkl\_gghgkmwlhjhly`_gghlt_fe_fh_ k\hc h^gZ nb]mjZ [hye`__gjZhy Z^yjmf]g__ F_jhc ijhkljZgkl_gghc ij y\e_lky iehsZ^v Q_f f_ggZnbj]mjyZz l_f f_gvr_ __ iehsZ^v b g

Ba dmjkZ]_hf_ljbb fu agZ_f dZd baf_jblv ^ebgm iehsZ qbke_ggu_ agZq_gby L_hjby \gZjhylghkkhl[uc ©hije_^gb_ª g_ \ kfuke_ jZkdjulby kZfh]h ihgylhygbZy \ qbke_ggh]h agZq_gby _jhylghklb \ l_konechnoj dhlhjuo **равновозможны** ?kebly o[u h^gh ba wlbo mkeh\bc g_ \uihegy_ nhjfmem deZkkbjqylkdhlklb_g_evay j_amevlZl [m^_l g_ijZ\bevg

?keb dhebq_kl\h \hafh`guo **бесконечно**, ij_`g_f **равновозможны,** lh qbke_ggh_ga_gZylghklb fh`gh gZclb ih nl ]_hf_ljbq_kdhydgh_kjhb $\frac{l_A}{l}$ beb $\frac{S_A}{S}$ beb $\frac{V_A}{V}$ \ aZ\bkbfhklb lh]h ]^_ e_`Zl lhqdb khhl_lkl\mxsb_ bkgoZh^eZbfgbwlbkigZbfe_gk beb \ lj_of_jghf ijhkljZgkl_

5. Типы случайных событий и действия над ними. Теоремы о вероятностях

Достоверное событие wlh kh[ulb_ dhlhjh_ h[ykZh^evgihjbijdhZ`^ ijh_^_gbb jZkkfZljb\Z_fh]h wdki_jbf_glZ

Невозможное событие wlh kh[ulb_ dhlhjh_ gb_dlh^jZbghicjbb ijh_^_gbb ^Zggh]h wdki_jbf_glZ Wlh agZqbl qlh kj_^b \k_ h^gh]h bkoh^ZZxsh[ehZ l_f k\hckl\hf dhlhjuf hij_^_e_ghkykh[u g_\hafh`guf qlh g_\hafh`ghfm kh[ulbcустое множество\mbkoh^h\ ^Zggh]h wdki_jbf_glZ GZijbf_j ijb [jhkZgbb ifbg[jhkZdblbh1 \uiZeb hj_je_rdZy\ey_lky g_\hafh`guf l d g_l bkoh^h\ ijb d hj_e b j_rdZ h^gh\j_f_ggh

Противоположное событие ih hlghr_gbx d jZkkfZhjhbkZbx wlh kh[ulb_ ¤dhlhjh_ g_ ijhbkoh^ihjhbkdhe^bl b gZh[hjhl Wlh a kh[ulbx khhl_lkl\m_ bkoh^u wdki_jbf_glZ dhlhju_ g_ khhl_ ijbq_f h[t_^bg_gb_ kmffZ bkoh^h\ khhl_lkl\mxsbZ jZ[gh iheghfm fgh`_kl\m \k_o bkoh^h\ ^Zggh]h wdki_jbf_glZ

>\Z kh[ulby gZau\Zxl **совместными,** _keb hgb fh]ml ijhba h^gh\j_f_ggh ijb h^ghf bkoh^_ **несовместными** wdki_jbf_glZ keb hgb g_ f ijhbahclb h^gh\j_f_ggh gb ijb hi^gbf_bkZh^l_ wdk khhl_lkl\m fgh`_kZo wdki_jbf_glh\ g_l h^bgZdh\uo h[sbo bkoh^h\

=hjZa^h [he__ keh`guf y\ey**независимости** kh[ulbc jZkkm`^_gby h lhf qlh ©gZklmie_gby bg_b_eh^gh]hZba g_aZ\bkb gbdZd g_ aZ\bkbl mie_l_gbZy beb g_ gZklmie_gby \lhjh]hª g_ m^h\e_l\hjbl_evgufb G_ bf_yj\h^afeh`yhwllh hngylb_ \iheg_ kl ]h\hjblv qlh ^\Zy:kh[u**считаются независимыми** _keb \_jhylghklv dZ` gbo b g_ aZ\bkbl hl gZklmie_gby beb g_ gZklmie_gby \l

Суммой ^\mo kemqZcguo kh[ulbcgZau\Zxl gh\hZ_cgbe kh[ulb_ dhlhjh_ ijhbkoh^bl _keb ijhbekoh^ey[eb[hh^gh\j_f_ggh Kh[u khhl_lkl\m_l hgb_^bgkmffZ fgh`_kl\ bkoh^h\ khhl_lkl\m b <

Произведением ^\mo kemqZcguo :kh[u<gZau\Z_lxl gkemqZcg kh[ulb_ dhlhjh_ ijhbkoh^bl lhevj^Z lhjh^Zkoh^yl kh[ulby h^gh\j_f_ggh Kh[ulbxkhhl_lkl\m_l i_j_k_q_gb_ fgh`_kl khhl_lkl\mxsbo kh[ulbyf

6. Обобщение умножения и сложения вероятностей

Определение 7. Kh[ulbyb gZau\Zxl _keb \ h^ghf b lhf bkiulZgbb ihy\e_gb_ h^gh]h ba gbo_gb_bkjeqZ_lxeny\eZdbo kijZ_^eb\Z ke_^mxsZy l_hj_fZ

Определение 8. ^\mo kh[ulbcgZau\Zxl kh[ulb_ dhlhjh_ khklhbl \ ihy\e_gbb eb[ehkh[ulby h^gh\j_f_ggh :gZeh]bqgh hij_^_ey_lky kmkfZ[ulbc khklhysZey_gbb ohl [u h^gh]h ba wlbo kh[ulbc

Теорема сложения. <_jhylghklv kmffy ^\mo**несовместных** kemqZcguo kh[ulb jZ\gZ kmff_ bo _jhylghkl_c >ey g_kh\f_klghuo kh[ulbcbf__f

?keb **<совместны** lh

?keb **события A и B зависимы** k mq_lhf nhjfmeu

Теорема умножения. <_jhylghklv ijhba_^ nZ[jbdb \uimkdZxl h^bgZdh\u_ eZfihqdb l_j\Zy **событий** : b jZ\gZ ijhba_b o g\b_xjhyklgc

?keh <**зависимы** lh

]^_ mkeh\gu_ _jhylghklb h^ggh kh_euvdy \hjh]h

РЕШЕНИЕ ЗАДАЧ

AZ^ZqZ>_ nZ[jbdb \uimkdZxl h^bgZdh\u_ eZfihqdb l_j\Zy eZfihq_d \lhjZy Kj_^b ijh^mdpbb i_j\hc ^_ndlgu_ kj_^b ijh\hmjhcb nZ[jbdb GZc^bl_ _jhylghklv lh]h kemqZcgh dm\ieZgZy eZfihqdb

Решение

lh mkeh\bx aZ^ZqnZ[jbdb eZfihq_d kh_euvdy \hjh]h keh\Zfb hgZ \uimkdZ_l <lhjZy nZ[jbdZ ZgZeh]bqgh \uimkdZ_l Kj_^b wlbo ih mkeh\bx [jZdZ qlh agZqbl ^_n_dlgufb k i_j\hc nZ[jbdb

:gZeh]bqgh gZc^_f ^hex ^_n_dlgu_ h[s_]h ijhba\h^kl\Z ^\mo <k_]h [jZdh\Zgguo eZfihq_d k h[_bo nZ[jbd [m^_l [m^_l jZ\gh _jhylghklb lh]h qlh kemqZcgh \uimkdZ_l i_j\uc aZ\h^ ^_n_dlghc

Hl_l

<_jhylghklv lh]h qlh eZfihqdb \uims_gu gZ i_j\hc nZ[jbdb

<_jhylghklv lh]h qlh eZfihqdb \uims_gu gZ \lhj [jZdh\Zggu_Â

lhwlhfm ih nhjfme_ iheghc _jhylghklb _jhylghklv lh]h \ fZ]Zabg_ kl_deh hdZ`_lky [jZdh\Zgguf jZ\gZ

Hl_l

AZ^ZqZ>\Z aZ\h^Z \uimkdZxl h^bgZdh\u_ Z\lhfh[bevgu_ ij_^h aZ\h\^uimkdZ_l ij_^hojZgbl_e_c \lhjhcuc aZ\h^ \uimkdZ_ [jZdh\Zgguo ij_^hojZgbl_e_c Z GZjch^bl_ _jhylghklv lh]h kemqZcgh \u[jZgguc \ fZ]Zabg_ ij_^hojZgbl_ev hdZ`_lky [jZd

Решение:

lh mkeh\bx aZ^Zqb i_j\uckdaZZ_\lh^ \uim_^hojZgbl_e_c b >jm]bfb keh\Zfb hg \uimkdZ_l <lhjhc aZ\h^ ZgZeh]bqgh \uimkdZ_l Kj_^b wlbo ih mkeh\bx [jZdZ hqlh agZvdlbljZ\gh Lh _klv hl \k_]h h[t_fZ \uims_gguo ^_lZe_c \uimkdZ_l i_j\uc aZ\h^

:gZeh]bqgh gZc^_f ^hex [jZdZ kh \lhjh]h aZ\h^Z <k_]h [jZdh\Zgguo ^_lZe_c k h[hbo aZ\h^h\ [m^_l Wlh b [m^_l jZ\gh _jhylghklb lh]h qlh kemqZcgh ij_^hojZgbl_ev hdZ`_lky [jZdh\Zgguf

Hl_l

AZ^ZqZGZ lj_gbjh\dm ijbr_e rdhevgbd kj_^bZgkbeb^\Zb [jZ\Z Rdhevgbdh\ ckeumqZ[jZahf ^_eyl gZ ljb nml[hevgu_ dhfZg^u

dZ`^hc GZc^bl_ _jhylghklv lh]h qlh <Zkbebc b IZ_e dhfZg^_

Решение:

GZk mkljhbl _keb <Zkbebc b IZ_e hdZ`mlky \ ex[hc ba lj_o dhfZg^

>himklbf kh[ulb_ h[Z ihiZebx \nml[hevgmx dhfZg^m

kh[ulb_ h<[Z ihiZebx\nml[hevgmx dhfZg^m

kh[ulb_ h<[Z ihiZebx \nml[hevgmx dhfZg^m

Wlb kh[ulby g_kh\f_klgu b ex[h_ ba gbo gZk mkljhbl J : J < J K

< k\hx hq_kjh_^ulb_ : khklhbl ba ^\mo aZ\bkbfuo kh[ulbc

: qlh <Zkbebc b IZ_e hdZ`_lky \ nml[hevghc dhfZg^_

: qlh IZ_e hdZ`_lky \ nml[hevghc dhfZg^_ J : :J

lh hij_^_e_gbx

gZoh^bf _jhylghklv qlh <Zkbebc b IZ_e \nml[hevgmx dhfZg^m P lZd dZd h^bg [eZ]hijbylguc bkoh^ Z Q lZd dZd \k_] lhwlhfm J :

L_i_jv gZc^_f Jlh _klv mkeh\gmx \h_jhylghklv lh]h qlh IZ_e \ x nml[hevgmx dhfZg^m \ mkeh\bc <Zkbebc b IZ_e m`_ ihiZe

Qbkeh [eZ]hijbylguo mkeh\bc jZ\gh dZd dZd_ ^Zgyfh <Zkbebl_f _klv gZk mkljZb\Z_l _keb IZ_e ihiZ^_l \ ex[h_ b dhfZgb^_P Z h[s__ qbkeh \k_o bkoh^h\ m`_ lZd dZd <Zk \u[hjd_ lh _klv \k_]h ij_l_g^_glh\ hklZehkv q_eh_d b Q lhwlhfm J

LZdbf h[jZahf J Â

:gZeh]bqgh jZkkm`^Zy gZc^bfJJK<

lhwlhfm J : < K

Hl_l

AZ^ZqZ]jhnbjfZ aZdmiZ_l dmjbgu_ ycpZ \ ^\mo ^hfZrgbo oha i_j\h]h ohayckl\ ycpZ \ukr_c dZl_]hjbb Z ba \lhjh]h ohayck \ukr_c dZl_]hjbb <k_]h \ukrmx dZl_]hjbx ihemqZ_l ybp GZ lh]h qlh ycph dmie_ggh_ m wlhc Z]jhnbjfu hdZ`_lky ba i_j

Пояснение.

Imklv kh[ulb_ A khklhbl \ lhf qlh ycph bf__l \ukrmx kh[ulby B_1 B_2 khklhyl \ lhf qlh ycph ijhba_^_gh \ i_j\hf b khhl_lkl_ggh Lh]^Z kh[ulby $A|B_1$ $A|B_2$ kh[ulby khklhysb_ \ lhf qlh yc dZl_]hjbb ijhba_^_gh \ i_j\hf b \lhjhf ohayckl_ khhl_lkl_ _jhylghklb _jhylghklv lh]h qlh [m^_l dmie_gh ycph \ukr_c

$$P(AB_1)+P(AB_2)=P(A|B_1)\cdot P(B_1)+P(A|B_2)\cdot P(B_2)=$$

$$=0,4\cdot P(B_1)+0,2\cdot(1-P(B_1))=0,2P(B_1)+0,2.$$

Ihkdhevdm ih mkeh\bx wlZ _jhylghklv jZ\gh _jhylghklb _jhylghklv lh]h qlh dmie_ggh_ ycph ijhba_^_gh \ i_j\hf ohayckl_ bf__f

$$P(B_1)=(0,35-0,2):0,2=0,75.$$

Примечание

Wlh j_r_gb_ fh`gh aZibkZlv dhjhldhkdhfZkylv _jhylghklv lh]h dmie_gh ycph ijhba_^_ggh_ \ i_j\hf ohayckl_ _jhylghklv lh]h dmie_gh ycph ijhba_^_ggh_ \h \lhjhf ohayckl_ _jhylghklb lhjhfme_

$$0,4x+0,2(1-x)=0,35\Leftrightarrow 0,2x=0,15\Leftrightarrow x=0,75.$$

Hl_l

Ijb_^_f ^jm]h_ j_r_gb_

lmklv \ i_j\hf ohayckl_ Z]jhnbyjbZ a\Zdmfi Z[_bkey_bp \ukr_c dZl_]hjbb Z \h \lhjhf² ohyabypckl_lhf qbkey_bp \ukr_c dZl_]hjbb kZfuf \k_]h Z]jhnhjfZ aZydmpiZ_\l lhf qbkey_bp \ukr_c dZl_]hjbb lh mkeh\bx \ukrmx dZl_]hjbx bf_xl ybp lh]^Z

$$\frac{0,4x+0,2y}{x+y}=0,35 \Leftrightarrow 0,4x+0,2y=0,35(x+y) \Leftrightarrow 0,05x=0,15y \Leftrightarrow x=3y.$$

Ke_^h\Zl_evgh m i_j\h]h ohayckl\Z aZdmiZxl \ ljb jZaZ lhwlhfm _jhylghklv lh]h qlh dmie_ggh_ ycph hdZ`_lky ba i_

$$\frac{3y}{3y+y}=\frac{3}{4}=0,75.$$

AZ^ZqZ< deZkk_ r_klbdeZkkgbd kj_^²b Fgbly ^b\Zl_^iym]ZdeZk kemqZcguf h[jZahf ^_eyl gZ ljb]qmeihu_dih\ dZ`^hc GZc \_jhylghklv lh]h qlh Fbly b l_ly hdZ`mlky \ jZaguo]jmiiZo

Решение.

lmklv h^bg ba ^jma_c gZoh^blky \ g_dhlhjhc]jmii_ hdZ`mlky q_eh\h_kdZbraboky h^ghdeZkkgbdh\ Z hklZevgu_ ]jmiZo <_jhylghklv lh]h qlh \lhjhc ^jm] hdZ`_lky kj_^b

Hl_l

AZ^ZqZ< kemqZcghf wdki_jbf_gl_ kbff_ljbqgmx fhg_lm [jh GZc^bl_ _jhylghklv lh]h
Z qlh gZkl mibl bkoh^ JHJ \ i_j\uc \hiZj^Zb_c jZradZ \h ²\lhjh hjze
[qlh \Z^_l ohly [u ^_ j_rdb

Решение.

<k_]h \hafh`guo bkohk^hv >\hbqgo **Обозначим 1-решка, 0-орел.**

	HHH
	HHJ
	HJH
	HJJ
	JHH
	JHJ
	JJH
	JJJ

: ;eZ]hijbylguf y\he^ybgkyj_hjdZe_rdZ

Ke_^h\Zl_evgh bkdhfZy _jhylghklv jZ\gZ

; ;eZhijbylgufb y\eyxlky q_luj_j_rjdZdZdZ j_rdZdZj_e j_rdZj_e_rdZ hjj_redZ_rdZ Ke_^h\Zl_evgh \bkhylghklv jZ

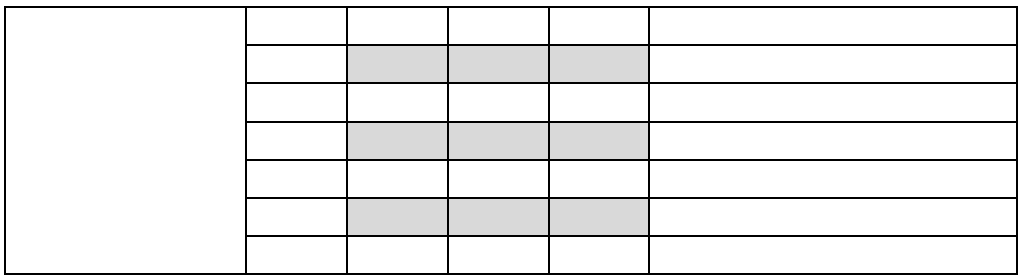
Hl_l

AZ^ZqZ< <her_[ghc kljZg_ [u\Z_l ^\Z lbiZ ih]h^u ohjhrZy b ih]h^Z mklZgh\b\rbkv mljhf ^_j`blky g_baf_gghc \_kv ^_ \_jhylghklvx ih]h^Z aZ\ljZ [m^_l lZdhc `_ dZd b k_]h^g <her_[ghc kljZg_ ohjhrZy GZc^bl_ _jhylghklv lh]h qlh hdly kljZg_ [m^_l hlebqgZy ih]h^Z

Решение.

JZkkfhljbf \k_\hafh`gu_ k0-m qiZbaa nomkaJ, **1-хорошая погода.**

	14.10	5.10	6.10	7.10	



>ey ih]h^u gZ b hdly[jy _klv \ZjbZglZ OOH O
² ohjhrZy² hlebqgZy ih]h^Z GZc^_f _jhylghklb gZklmie_gb
3 ;;2
3 ;22
3 2;2
3 222
MdZaZggu_ kh[ulby g_kh\f_klgu_ _jhylghklv blo kh\f_u
wlbo kh[ulbc
3 OOH3 OHH3 HOH3 HHH
Hl_l

AZ^ZqZ

< <her_[ghc kljZg_ [u\Z_l
lbiZ ih]h^u ohjhrZy b hleb
ijbqzf ih]h^Z mklZgh\b\
mljhf ^_j`blky g_baf_gghc
^_gvBa_klgh qlh k _jhylgh
ih]h^Z aZ\ljZ [m^_l lZdh
dZd b k_]h^gy K_]h^gy bx
\ <her_[ghc kljZg_ ohjh
GZc^bl_ _jhylghklv lh]h q
bxey \ <her_[ghc kljZg_
hlebqgZy ih]h^Z

Решение:

\ZjbZglZ OOH OHH HOH HHH
3 OOH3 OHH3 HOH3 HHH Â Â Â ÂÂ Â Â Â

Hl_l

AZ^ZqZ< lhj]h\hf p_glj_ ^\Z h^bgZdh\u [redacted]
ijh^Zxl dhn_ <_jhylghklv lh]h qlh d
Z\lhfZl_ aZdhgqblky dhn_ jz\jZ Z\jhylgh
lh]h qlh dhn_ aZdhgqblky \ h[hbo Z
GZc^bl_ _jhylghklv lh]h qlh d
hklZg_lky \ h[hbo Z\lhfZlZo

Решение : ±dhn_ aZdhgqblky \ i_j\hf Z\lh ±dhn
aZdhgqblky \h \lhjhf Z\lhfZl_
lh mkeh\bx aZ^Zqb

(+)= + − =
<_jhylghklv ijhlb\hiheh`gh]h kh[ulby ©dhn_ hklZg_lky \
(+)= − =
Hl_l

AZ^ZqZ <_jhylghklb ihjZ`_gby p_eb i_j\uf b \lhjuf hjm^
khhl_lkl_ggh b GZclb _jhylghklv ihjZ`_gby p_eb ihjZ`_gZei_
Решение lhkdhevdm _jhylghklb ihjZ`_gby p_eb hjm^byfb
khhl_lkl_ggh g_ aZ\bkyl hl j_amevlZlh\ klj_ev[u dZ`^h]h b
g_aZ\bkbfu BkdhfZy _jhylghklv jZkkqblu\Z_lky ih nhjfme_

H[Z hjm^by ihjZayl p_ev

AZ^ZqZ l_j_^ gZqZehf i_j\h]h lmjZ q_fibhgZlZ ih rZofZlZ
jZa[b\Zxl gZ b]jh\u_ iZju kemqZcguf h[jZahf k ihfhsvx
q_fibhgZl_ mqZkl\mxl rZofZlbklh\ kj_^b dhlhjuo ij_^klZ
lhf qbk_e<Zkbebc AZcp_\ GZc^bl_ _jhylghklv lh]h qlh \ i_
AZcp_\ [m^_l b]jZlvek[u dZ`^hf rZofZlbklhf ba Jhkkbb
Решение:

< i_j\hf lmj_ mqZkl\mxl rZofZlbklh\ hgb jZa[blu g

>himklbf <Zkbebc AZcpi_\jhdZ`_lky rZofZlbklh\ <_jhylgh
gZc^_f ih hij_^_e_gbx _jhylghklb
<_jhylghklv lh]h qlh \lhjuf hdZ`_lky jhkkbygbg ihkqblZ
jhylghklb lZd dZd [eZ]hijbZlgu]h \lhjuf \gh bkoh
lZd dZd <Zkbebc m`_ \u[jZg

AgZqbl _jhylghklv lh]h qlh <Zkbebc AZcp_\ hdZ`_lky
jZ\gZ
ih nhjfme_ ihbkdZ _jhylghklb ijhba_

Gh aZf_lbf qlh gZk mkljhbeh [u qlh <Zkbebc [m^_l \
\lhjuf jhkkbygbghf lhfm bkdhfZy _jhylghklv [m^_l jZ\g
ih nhjfme_
_jhylghklb kmffu g_kh\f_klguo kh[ulbc
L_hj_fZ <_jhylghklv kmffu g_kh\f_klguo kh[ulbc :
_jhylghklb wlco kh[ulbc< J : J <
Hl_l

AZ^ZqZ g_ ^himklblv >himklbf qlh jZa ih^[jZku\ZeZkv fh
`^uc jZa \uiZ^Ze hj_e DZdh\Z _jhylghklv lh]h qlh ijb gh
hj_e"
Решение.

<_jhylghklv \uiZ^_gby hjeZ ijb h^ghf [jhkZgbb fhg_lu jZ
<_jhylghklv \uiZ^_gby hjeZ \ dZ`^hf ba iylb [jhkZgbc_ i
hq_gv g__ebdZ Ijbgylh qlh _jhylghklv \uiZ^Zgby hjeZ ijb
wlh agZqbl qlh \gh\^ufh`gu b ijb dZ`^hf [jhkZgbb fhg_l

\uiZ^_gby hklZ_lky ihklhygghcg_ aZjZb\ghfh hl j_amevlZlh\
[jhkZgbc
Hl_l

AZ^ZqZGZc^bl_ _jhylghklv lh]h qlh keamhfZ\cg[uZghjZ ^\magZ
qbkeh ^j_e_gbb gZ ^Z_l \ hklZld_ Hdjm]eblv ^h lukyqguo
Решение.
H[s__ dhebq_kl\h ^\magZqguo qbk_eu[h^jhdZ qbkeZ kqbl
jZ\gh\hafh`guf
JZkkfhljbf kh[ulb_ ©kemqZcguf h[jZahf \u[jZggh_ ^\magZ
^_e_gbb gZ ^Zd_l \ªhklZ
?keb qbkeijb ^_e_gbb gZ ^Z_l \ hklZld_ ij_^lhlZ\jhlvfh\
\b^_ G_h[oh^bfh i_j_kqjblZdvlhjuhklZ_lky ^\magZqguf
[m^_l dhebq_kl\h[eZ]hijbylkl\mxsbo khi[u^lbx^yl
lh _klv Bkdhfzy _jhylghklv j$\frac{m_A}{n}$\g$\frac{7}{90}$s
Hl_l

AZ^ZqZGZc^bl_ _jhylghklv lh]h qlh keamhfZ\cg[uZghjZ ^\magZ
qbkeh ijb ^_e_gbb gZ ^Z_l \ hklZld_
Решение.
H[s__ qbkeh ^\magZqguo
Kh[ulb_ ©kemqZcguf h[jZahf \u[jZggh_ ^\magZqgh_ qbk
^Z_l \ hklZld_ a
Dhebq_kl\h [eZ]hijbylkl\mxsbjZ\gh\hoh\hkemqahgZb dhlhjuo
qbkehN ^\magZqgh_ Wlh [m^_l ijb lh _klv
Bkdhfzy _jhylghklv $\frac{m_A}{n}=\frac{9}{90}=\frac{1}{10}$
Hl_l

AZ^Zgb_Klj_ehd klj_ey_l ih fbr_gb h^bg jZa < kemqZ_ ijhfZ
\lhjhc \uklj_e ih lhc `_ fbr_gb <_jhylghklv ihiZklv \ fbr_
\uklj_e_ jZ\gZ GZc^bl_ _jhylghklv lh]h qlh fbr_gv [m^
i_j\uf eb[h \lhjuf \uklj_ehf
Пояснение.
Imklv² kh[ulb_ khklhys__ \ lhf qlh fbr_gv ihjZ`_gZ k
\uklj_eZ² kh[ulb_ khklhys__ \ lhf qlh fbr_gv ihjZ`_gZ k
<_jhylghklv khiZklvZ Kh[ulb_klmiZ_l _keb klj_eyy i
klj_ehd ijhfZogmeky Z klj_eyy \lhjhc jZa ihiZe Wlh
jhylghklv jZ\gZ ijhba^_gbx _jhylghkl_c wlbo kh[ulbc
Kh[ulbyb g_kh\f_klgu_ _jhylghklv bo mfffu_jZ\gZghkl_c v
kh[ulbc
Hl_l

AZ^Zgb_Dh\[hc >`hg ihiZ^Z_l \ fmom gZ kl_g_ k _jhylghklvx
ba ijbklj_eyggu_ Dh\[hc >`hg klj_ey_l ba g_ijbklj_ey
lh hg ihiZ^Z_l \ fmom k _jhylghklvx GZ klhe_ e_`bl
lhevdh ijbklj_eyggu_ Dh\[hc >`hg \b^bl gZ kl_g_ fmom g
ihiZ\rbcky j_\hev_jeyb_klj_fmom GZc^bl_ _jhylghklv lh]h
ijhfZogzlky
Пояснение.

>`hg ijhfZog_lky _keb ko\Zlbl ijbklj_eygguc j_\hev_j beb _keb ko\Zlbl g_ijbklj_eygguc j_\hev_j b ijhfZog_lky ba _jhylghklb _jhylghklb wlbo kh[ulbc jZ\gu khhl_ílkl_ggh b í Wlb kh[ulby g_kh\f_klgu _jhylghklv bo kr _jhylghkl_c wlbo kh[ulbc

Hl_l

Ijb_^_f ^jm]h_ j_r_gb_

>`hg ihiZ^Z_l \ fmom\Zlbl ijbklj_eygguc j_\hev\_j b ihiZ beb _keb ko\Zlbl g_ijbklj_eygguc j_\hev\_j b ihiZ^Z_l ba g \_jhylghklb \_jhylghklb wlbo kh[ulbc jZ\gu khhl\_lbkl\_ Wlb kh[ulby g_kh\f_klgu \_jhylghklv bo kmff_ jZ\gZ kmff_ \_ wlbo kh[ulbc Kh[ulb_ khklhys__ \ lhf qlh >`hg ijhlb\hiheh`gh_ ?]h _jhylghklv jZ\gZ

AZ^Zgb_Ba ysbdZ \ dhlhjhf e_`Zl nehfZkl_ju g_]ey^y nehfZkl_jZ GZc^bl_ _jhylghklv lh]h qlh wlb nehfZkl_ju p_lZ _keb ba_klgh qlh \ ysbd_ kbgbo b djZkguo neh

Решение.

BagZqZevgh \ ysbd_ nehfZkl_jh\ Ijb \u[hj_ i_j\h]h r lh]h qlh hg [m^]h kb_lZ jZ\gZ ?keb i_j\uc \u[jZg ^_ckl\bl_evgh hdZaZeky kbg_]h p_lZ lh \ ysbd_ hklZ_lky lhwlhfm _jhylghklv \u[hjZ \lhjh]h kbg_]h nehfZkl_jZ jZ\gZ \lhjhc nehfZkl_ju \u[hjZ kbgbfb ihemqZ_f ijhba_^_gb_ wlbo \

$$P_1 = \frac{12}{25} \cdot \frac{11}{24} = \frac{132}{600} = 0,22$$

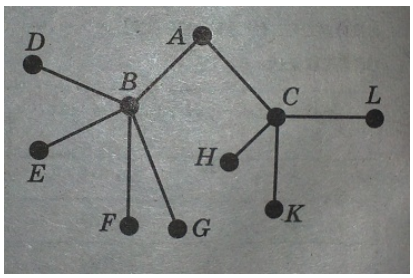
JZkkfhljbf \lhjmx kblmZpbx dh]^Z h[Z nehfZkl_jZ fh]ml [ul jZkkm`^_gbyfb ihemqZ_f _jhylghklv

$$P_2 = \frac{13}{25} \cdot \frac{12}{24} = \frac{13}{50} = 0,26$$

LZd dZd gZk bgl_j_km_l gZklmbe_gbl_jhjh bkjh^Z ijb g_kh\ wlbo kh[ulbc ihemqZ_f agZq_gb_ bkdhfhc _jhylghklb

$$P = P_1 + P_2 = 0,22 + 0,26 = 0,48$$

Hl_l



AZ^ZqZ l_gkbhg_j]mey_l ih ^hjh`dZf iZ dZ`^hc jZa\bed_ hg gZm^Zqm \u[bjZ_ ^hjh`dm g_ \ha\jZsZykv h[jZlgh Ko_ ihdZaZgZ gZ jbkmkbdhg_j gZqbgZ_l ijh]m lhqd_ : GZc^bl_ _jhylghklv lh]h qlh h lhqdm *

Решение:

Qlh[u ihiZklv \ lhqdm *^hie`gkbhg_j kgZqZeZ ihiZklv \lhqdm <_keb hg kgZqZ \ lhqdm K lh ih mkeh\bx aZ^jZqh\_jhmqlv $ hg g_ kfh`_l

lhwlhfm gZihfgbf qlh lZdh_ _jhylghklv

$$P_1+P_2+P_3=\frac{1}{8}+\frac{1}{8}+\frac{1}{16}=\frac{5}{16}=0,3125$$

Hl_l

AZ^ZqZAZ dejmc klhe gZ e \kmeZchf ijhy^d_ jZZb\Zxlky fZev qbdh\ b ^_\hqdb GZylghklv lh]h qlh f_\hqdZb^_[m^_l^_kbv h^bg fZevqbd

Решение.

lmklv i_j\hc aZ klhe kyh]^Z^_\hqdZ`^h]h ba hklZ\rboky qbke_ b ^ey \lhjhc ^_\hqdb _jhylghklv hdZaZl $\frac{1}{100}$ y gZ ex[h klmev_\ j_\gZ

: f_kl m^h\e_l\hjyxsbo mkeh\bx aZ^Zqb lhevdh ^\Z LZ qlh f_`^m ^\mfy ^_\hqdZfb [mbg l kh^ lv fZevqbd jZ\gZ

$$2\cdot\frac{1}{100}=0,02$$

Hl_l

Bkihevam_fZy ebl_jZlmjZ

AZ^Zqb hldjulh]^Zbcg?=WZlfZbd_

< G Klm^_g_pdZy ©J_r_gb_ aZ^Zq ih klZlbklbd_ _jhylghkl_cª <he]h]jZ^]

©?=W fZl_fZlbdZ Ysª_gbh< [Zah\uc ijhnbevguc mjh_ Bgl_jg±l_kmjku

> > =msbg J_rm ?=W]